```
→ /* Fourier Analysis on the Euclidean space
1. Fourier transform of the Gaussian function
2. Load the package for Fourier series
3. Fourier series of x-[x+1/2]
4. Gibbs phenomenon for x-[x+1/2]
5. Fourier series of the sawtooth function
6. Uniform convergence of the Fourier series of the sawtooth function
*/
```

```
(%i2) /* 1. Fourier transform of the Gaussian function */
integrate(exp(-2 · %pi · %i · x · ξ-%pi · x^2),x,-inf,inf);
integrate(exp(-%i · x · ξ-x^2/2),x,-inf,inf)/sqrt(2 · %pi);
```

(%o1) $\%e^{-\pi\,\xi^2}$

(%o2) $\%e^{-\frac{\xi^2}{2}}$

```
(%i3) /* 2. Load the package for Fourier series
   The spelling of the package is "fourie" and not "fourier". */
load("fourie")$;
```

```
(%i4) /* 3. Fourier series of x-1/2-[x-1/2]
   totalfourier(function,variable of function,period/2) */
totalfourier(x,x,1/2);
```

(%t4) $a_0 = 0$

(%t5) $a_n = 0$

(%t6) $b_n = 4\left(\frac{\sin(\pi n)}{4\,\pi^2\,n^2} - \frac{\cos(\pi n)}{4\,\pi\,n}\right)$

(%t7) $a_0 = 0$

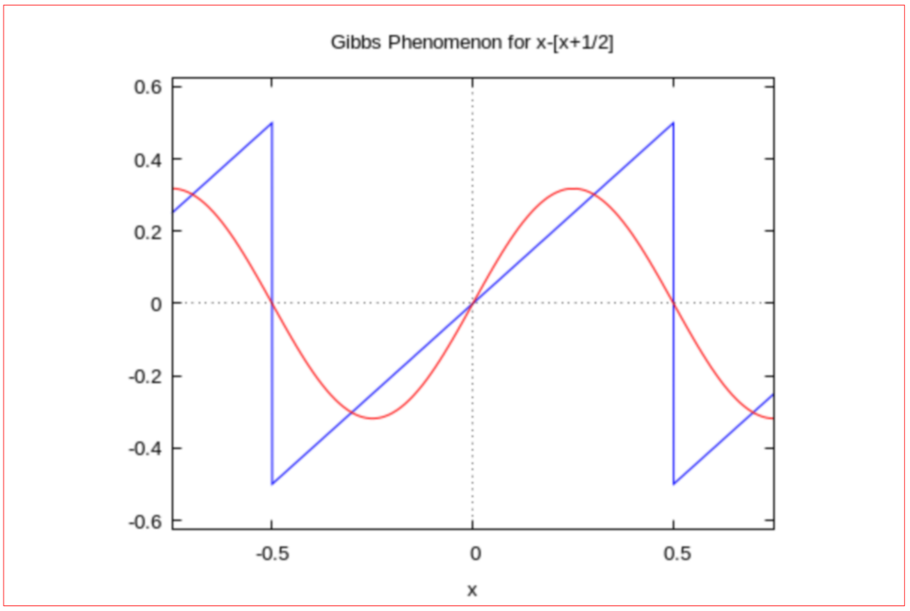
(%t8) $a_n = 0$

(%t9) $b_n = -\frac{(-1)^n}{\pi\,n}$

(%o9) $-\frac{\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin(2\,\pi\,n\,x)}{n}}{\pi}$

(%i10)
```
/* 4. ANIMATION
The partial sum of the first n terms
of the Fourier series of  the frac function x-[x+1/2]
for N=1,2,3,...,50 */
wxanimate(N, 50,
[x-floor(x+1/2),sum(-(-1)^n*sin(2*%pi*n*x)/(n*%pi),n,1,N)],
[x,-3/4,3/4],[y,-5/8,5/8],
[legend,false],[xtics, -1/2,1/2,1/2],
[title,"Gibbs Phenomenon for x-[x+1/2]"]),
    wxanimate_framerate=5;
```

(%t10)



(%o10)

(%i11) /* 5. Fourier series of the sawtooth function
totalfourier(function,variable of function,period/2) */
```
totalfourier(1/2-abs(x),x,1/2);
```

(%t11) $a_0 = \frac{1}{4}$

(%t12) $a_n = 4\left(\frac{1}{4\,\pi^2\,n^2} - \frac{\cos(\pi\,n)}{4\,\pi^2\,n^2}\right)$

(%t13) $b_n = 0$

(%t14) $a_0 = \frac{1}{2^2}$

(%t15) $a_n = -\frac{(-1)^n - 1}{\pi^2\,n^2}$

(%t16) $b_n = 0$

(%o16) $$\frac{1}{4} - \frac{\sum_{n=1}^{\infty} \frac{((-1)^n - 1)\cos(2\,\pi\,n\,x)}{n^2}}{\pi^2}$$

(%i17) /* 6. ANIMATION
The partial sum of the first n terms
of the Fourier series of the sawtooth function
for N=1,2,3,...,50 */

```
wxanimate(N, 50,
[1/2-abs(x-floor(x+1/2)),1/4+sum((1-(-1)^n) · cos(2 · %pi · n · x)/(n^2 · %pi^2),n
[x,-3/4,3/4],[y,-1/8,5/8],
[legend,false],[xtics, -1/2,1/2,1/2],
[title,"Uniform convergence of the Fourier series of the sawtooth function"]),
    wxanimate_framerate=5$
```

(%t17)