→ /·
– a linear system without solutions
– a linear system with infinitely many solutions
– determinant and inverse
·/

→ /·
a linear system without solutions
·/

(%i1) /· Use the command solving linear systems ·/
```
linsolve([x1-x3-2*x5=1, x2+x3+x5=-2, -x1+x3+x4+x5=3, 2*x1+x2-x3-3*x5=1],
```
(%o1) []

→ /· The output [] means "no solution". ·/

(%i2) /· Input the augmented matrix A ·/
```
A:matrix(
 [1,0,-1,0,-2,1],
 [0,1,1,0,1,-2],
 [-1,0,1,1,1,3],
 [2,1,-1,0,-3,1]
);
```
(A)
$$\begin{pmatrix} 1 & 0 & -1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 & -2 \\ -1 & 0 & 1 & 1 & 1 & 3 \\ 2 & 1 & -1 & 0 & -3 & 1 \end{pmatrix}$$

(%i3) /· Use the command of row operation for reduction
(just one step before the complete row reduction) ·/
```
echelon(A);
```
(%o3)
$$\begin{pmatrix} 1 & 0 & -1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(%i4) /· Use the command computing the rank ·/
```
rank(A);
```
(%o4) 4

(%i5) /* Use the command showing the transpose */
transpose(A);

(%o5)
$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ -1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ -2 & 1 & 1 & -3 \\ 1 & -2 & 3 & 1 \end{pmatrix}$$

→ /*
a linear system with infinitely many solutions
*/

(%i6) /* Use the command solving linear systems */
linsolve([x1-2*x2+3*x4=2, x1-2*x2+x3+2*x4+x5=2, 2*x1-4*x2+x3+5*x4+2*x

(%o6) $[x1=-3\,\%r2+2\,\%r1+2,\ x2=\%r1,\ x3=\%r2-1,\ x4=\%r2,\ x5=1]$

(%i7) /* The output shows that solutions are given as follows */
x
=
matrix([2],[0],[-1],[0],[1])
+
%r1*matrix([2],[1],[0],[0],[0])
+
%r2*matrix([-3],[0],[1],[1],[0]);

(%o7)
$$x=\begin{pmatrix} -3\,\%r2+2\,\%r1+2 \\ \%r1 \\ \%r2-1 \\ \%r2 \\ 1 \end{pmatrix}$$

(%i8) /* Input the argumented matrix B */
B:matrix(
 [1,-2,0,3,0,2],
 [1,-2,1,2,1,2],
 [2,-4,1,5,2,5]
);

(B)
$$\begin{pmatrix} 1 & -2 & 0 & 3 & 0 & 2 \\ 1 & -2 & 1 & 2 & 1 & 2 \\ 2 & -4 & 1 & 5 & 2 & 5 \end{pmatrix}$$

(%i9)
```
/* Use the command of row operation for reduction
 (just one step before the complete row reduction) */
echelon(B);
```

(%o9)
$$\begin{pmatrix} 1 & -2 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

(%i10)
```
/* Use the command computing the rank */
rank(B);
```

(%o10) 3

→
```
/*
determinant and inverse
*/
```

(%i11)
```
/* Input a square matrix C */
C:matrix(
 [3,5,1,2,1],
 [2,6,0,9,3],
 [3,6,7,1,2],
 [2,7,0,0,0],
 [1,5,0,0,0]
);
```

(C)
$$\begin{pmatrix} 3 & 5 & 1 & 2 & 1 \\ 2 & 6 & 0 & 9 & 3 \\ 3 & 6 & 7 & 1 & 2 \\ 2 & 7 & 0 & 0 & 0 \\ 1 & 5 & 0 & 0 & 0 \end{pmatrix}$$

(%i12)
```
/* Use the command computing the determinant */
determinant(C);
```

(%o12) −18

(%i13)
```
/* Use the comand computing the inverse */
invert(C);
```

(%o13)
$$\begin{pmatrix} 0 & 0 & 0 & \frac{5}{3} & -\frac{7}{3} \\ 0 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} \\ -\frac{5}{2} & \frac{1}{2} & \frac{1}{2} & \frac{37}{6} & -\frac{22}{3} \\ -\frac{7}{2} & \frac{5}{6} & \frac{1}{2} & \frac{163}{18} & -\frac{97}{9} \\ \frac{21}{2} & -\frac{13}{6} & -\frac{3}{2} & -\frac{497}{18} & \frac{293}{9} \end{pmatrix}$$